Superconducting diode effect

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[1a] Ya.V. Fominov, D.S. Mikhailov, *Asymmetric higher-harmonic SQUID as a Josephson diode*, Phys. Rev. B 106, 134514 (2022).

[1b] **G.S. Seleznev**, Ya.V. Fominov, *Influence of capacitance and thermal fluctuations on the Josephson diode effect in asymmetric higher-harmonic SQUIDs*, Phys. Rev. B 110, 104508 (2024).

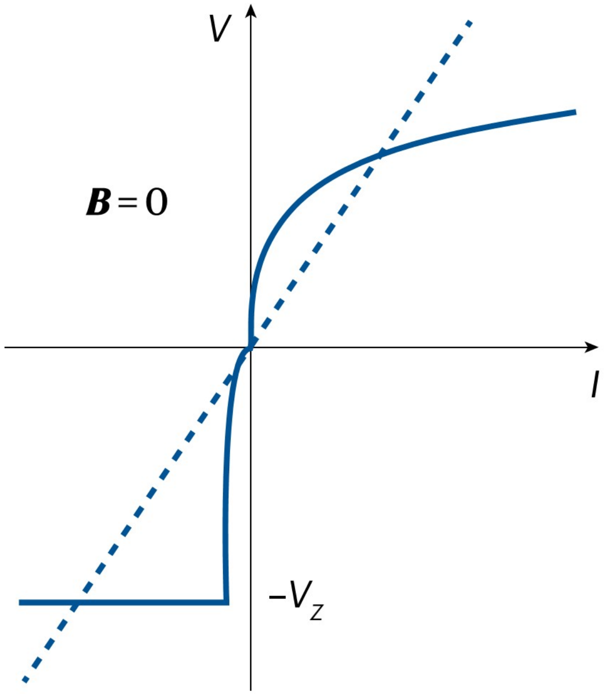
[2] D.S. Kalashnikov, **G.S. Seleznev**, A. Kudriashov, Y. Babich, D.Yu. Vodolazov, Ya.V. Fominov, V.S. Stolyarov, *Diode effect in Shapiro steps in an asymmetric SQUID with a superconducting nanobridge*, in preparation.

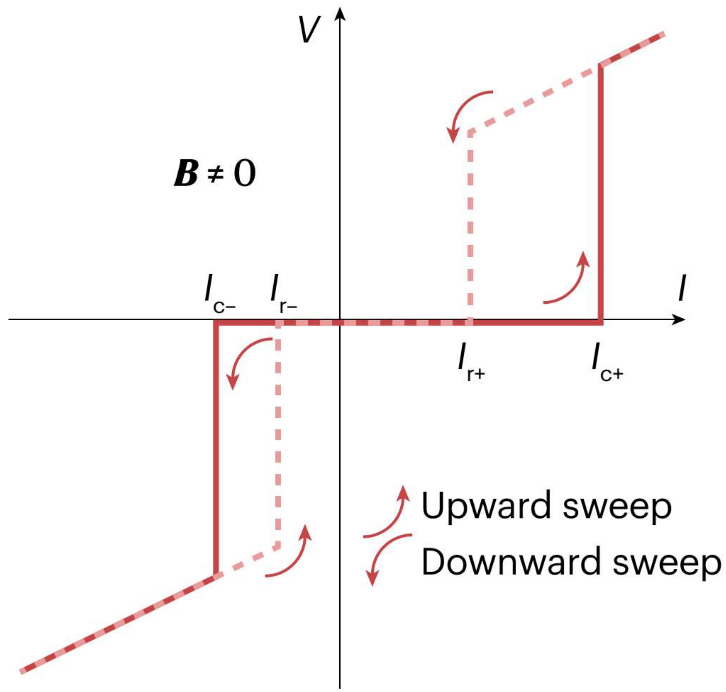
[3] **Yu.A. Dmitrievtsev**, Ya.V. Fominov, *Superconducting orbital diode effect in SN bilayers*, in preparation.

23 June 2025

Diode

Normal vs Superconducting





[from review Nadeem *et al.*, Nat. Rev. Phys. (2023)]

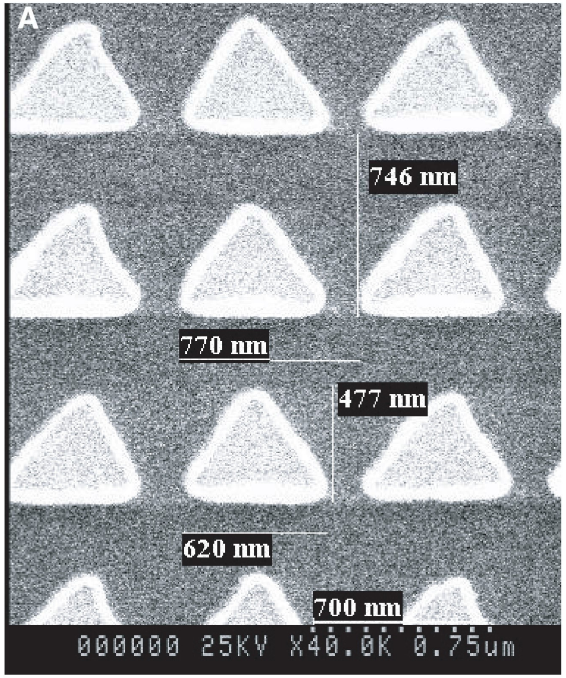
Superconducting ratchet effect: asymmetric pinning

Science **302**, 1188 (2003)

A Superconducting Reversible Rectifier That Controls the Motion of Magnetic Flux Quanta

J. E. Villegas,[1] Sergey Savel'ev,[2] Franco Nori,[2,3] E. M. Gonzalez,[1] J. V. Anguita,[4] R. García,[4] J. L. Vicent[1]*

We fabricated a device that controls the motion of flux quanta in a niobium superconducting film grown on an array of nanoscale triangular pinning potentials. The controllable rectification of the vortex motion is due to the asymmetry of the fabricated magnetic pinning centers. The reversal in the direction of the vortex flow is explained by the interaction between the vortices trapped on the magnetic nanostructures and the interstitial vortices. The applied magnetic field and input current strength can tune both the polarity and magnitude of the rectified vortex flow. Our ratchet system is explained and modeled theoretically, taking the interactions between particles into consideration.



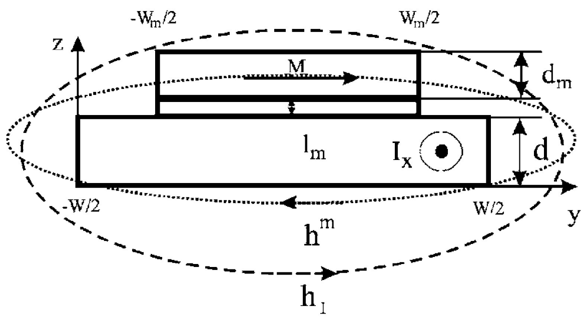
Superconducting diode effect: stray-field effect

PHYSICAL REVIEW B **72**, 064509 (2005)

Considerable enhancement of the critical current in a superconducting film by a magnetized magnetic strip

D. Y. Vodolazov,* B. A. Gribkov, S. A. Gusev, A. Yu. Klimov, Yu. N. Nozdrin, V. V. Rogov, and S. N. Vdovichev†
Institute for Physics of Microstructures, Russian Academy of Sciences, 603950 Nizhny Novgorod, GSP-105, Russia
(Received 8 April 2005; revised manuscript received 21 June 2005; published 9 August 2005)

We show that a magnetic strip on top of a superconducting strip magnetized in a specified direction may considerably enhance the critical current in the sample. At fixed magnetization of the magnet we observed the diode effect—the value of the critical current depends on the direction of the transport current. We explain these effects by a influence of the nonuniform magnetic field induced by the magnet on the current distribution in the superconducting strip. The experiment on a hybrid Nb/Co structure confirmed the predicted variation of the critical current with a changing value of magnetization and direction of the transport current.



Superconducting diode effect: asymmetry due to domain wall

J. Phys.: Condens. Matter **26** (2014) 095702 (7pp) doi:10.1088/0953-8984/26/9/095702

The diode effect induced by domain-wall superconductivity

M A Silaev[1], A Yu Aladyshkin[1,2], M V Silaeva[3] and A S Aladyshkina[3]

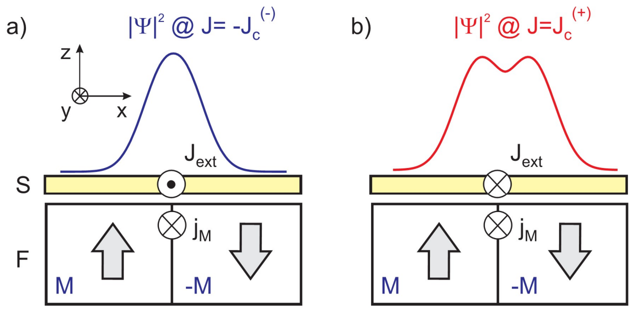


Figure 1. Schematic view of the hybrid system, consisting of a superconducting film (S) and ferromagnetic (F) substrate with a single domain wall. The grey arrows show the local magnetization and the orientation of the magnetic self-field above the magnetic domains. The parity-breaking vector $\mathbf{j}_M$ given by equation (1) is oriented along the y-axis; the direction of the external current is antiparallel to $\mathbf{j}_M$ (panel a) and parallel to $\mathbf{j}_M$ (panel b). The OP profiles $|\Psi|^2$ for the DWS channel, carrying maximal supercurrents in each direction, are shown schematically.

$$\mathbf{M} = M_z(x)\hat{\mathbf{z}}$$

Parity-breaking vector:

$$\mathbf{j}_M = c\,\nabla \times \mathbf{M}$$

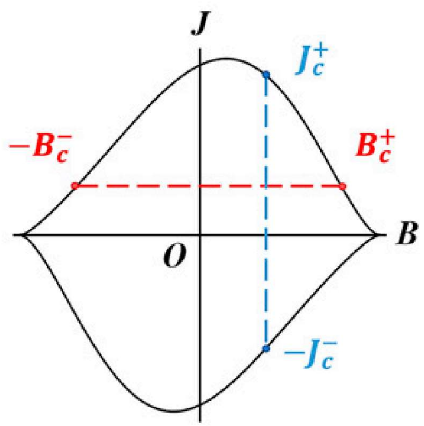
Superconducting diode effect: spin-orbit coupling + Zeeman field

PNAS **119**, e2119548119 (2022):

Supercurrent diode effect and finite-momentum superconductors

Noah F. Q. Yuan[a,1] and Liang Fu[b,1]

When both inversion and time-reversal symmetries are broken, the critical current of a superconductor can be nonreciprocal. In this work, we show that, in certain classes of two-dimensional superconductors with antisymmetric spin–orbit coupling, Cooper pairs acquire a finite momentum upon the application of an in-plane magnetic field, and, as a result, critical currents in the direction parallel and antiparallel to the Cooper pair momentum become unequal. This supercurrent diode effect is also manifested in the polarity dependence of in-plane critical fields induced by a supercurrent. These nonreciprocal effects may be found in polar $SrTiO_3$ film, few-layer $MoTe_2$ in the T_d phase, and twisted bilayer graphene in which the valley degree of freedom plays a role analogous to spin.



$\Delta(\mathbf{r}) = \Delta_0 e^{i\mathbf{Q}\mathbf{r}}$ (helical state)

Ingredients:
- broken inversion symmetry (P-symmetry) – spin-orbit coupling
- broken time-reversal symmetry (T-symmetry) – magnetic (Zeeman) field

Superconducting diode effect

Magnetostatics of superconductors without an inversion center

L. S. Levitov, Yu. V. Nazarov, and G. M. Éliashberg
L. D. Landau Institute of Theoretical Physics, Academy of Sciences of the USSR

(Submitted 7 January 1985)
Pis'ma Zh. Eksp. Teor. Fiz. **41**, No. 9, 365–367 (10 May 1985)

The Ginzburg–Landau equation for superconductors of polar symmetry

J. Phys.: Condens. Matter **8** (1996) 339–349.

Victor M Edelstein

direction of **Q**

$\delta = \alpha_{\text{so}} p_F / E_F$

Critical current in a thin film:

$$J_c(B) = J_c(0)\left[1 + (\boldsymbol{c}\times\boldsymbol{B})\cdot\hat{\boldsymbol{J}}\, f_3\, \delta\, \frac{3(7\zeta(3))^{1/2}}{8H_{c2}\,p_F\,\xi(T)}\right]$$

(due to presence of the Lifshitz invariant terms in the GL free energy)

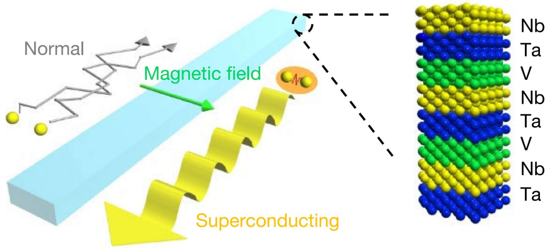
Superconducting diode effect (experiment)

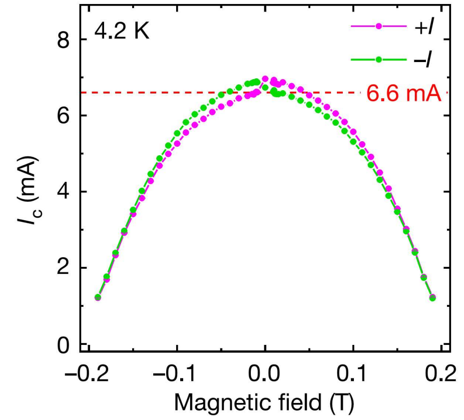
Observation of superconducting diode effect

https://doi.org/10.1038/s41586-020-2590-4
Received: 14 March 2020

Fuyuki Ando[1], Yuta Miyasaka[1], Tian Li[1], Jun Ishizuka[2], Tomonori Arakawa[3,4], Yoichi Shiota[1], Takahiro Moriyama[1], Youichi Yanase[2] & Teruo Ono[1,4]

[Nature (2020)]



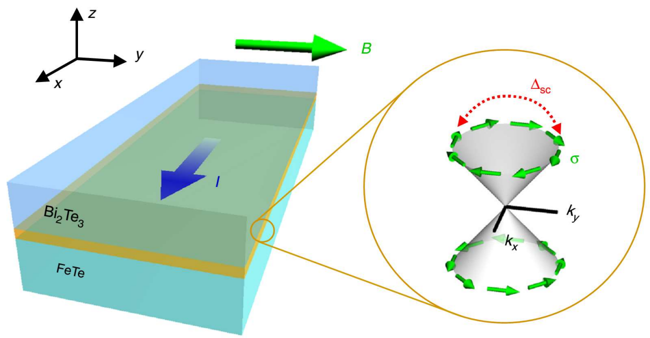


Nonreciprocal charge transport at topological insulator/superconductor interface

Kenji Yasuda[1,5], Hironori Yasuda[1], Tian Liang[2], Ryutaro Yoshimi[2], Atsushi Tsukazaki[3], Kei S. Takahashi[2], Naoto Nagaosa[1,2], Masashi Kawasaki[1,2] & Yoshinori Tokura[1,2,4]

[Nature Communications (2019)]

$$R(I) = R_0\big(1 + \gamma\hat{\mathbf{z}}(\mathbf{B}\times\mathbf{I})\big)$$



Magnetoelectric anisotropy [Rikken *et al.* (2001), (2005)]

strongly enhanced by superconducting fluctuations near T_c : $\dfrac{\gamma_S}{\gamma_N} \sim \left(\dfrac{E_F}{T_c}\right)^3$ [Wakatsuki *et al.* (2017)]

Second-harmonic voltage (resistance) measurements: $V = R_0 I + \gamma R_0 B I^2$ ➡ $R_\omega = R_0, \quad R_{2\omega} = \dfrac{R_0}{2}\gamma B I_0$

$I = I_0 \sin\omega t$

Josephson diode effect (JDE)

PHYSICAL REVIEW B **89**, 195407 (2014)

Anomalous Josephson effect induced by spin-orbit interaction and Zeeman effect in semiconductor nanowires

Tomohiro Yokoyama Mikio Eto Yuli V. Nazarov

PHYSICAL REVIEW B **103**, 144520 (2021)

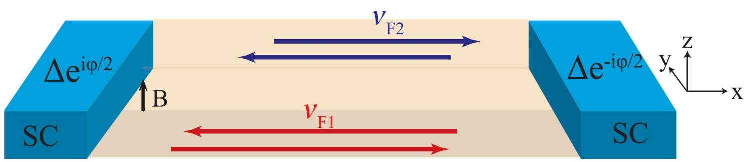
Geometry controlled superconducting diode and anomalous Josephson effect triggered by the topological phase transition in curved proximitized nanowires

A. A. Kopasov,[1,2,*] A. G. Kutlin,[3] and A. S. Mel'nikov[1,2,4]

Asymmetric Josephson effect in inversion symmetry breaking topological materials

Chui-Zhen Chen,[1] James Jun He,[1] Mazhar N. Ali,[2] Gil-Ho Lee,[3,*] Kin Chung Fong,[4,†] and K. T. Law[1,‡]

PHYSICAL REVIEW B **98**, 075430 (2018)



Ingredients:

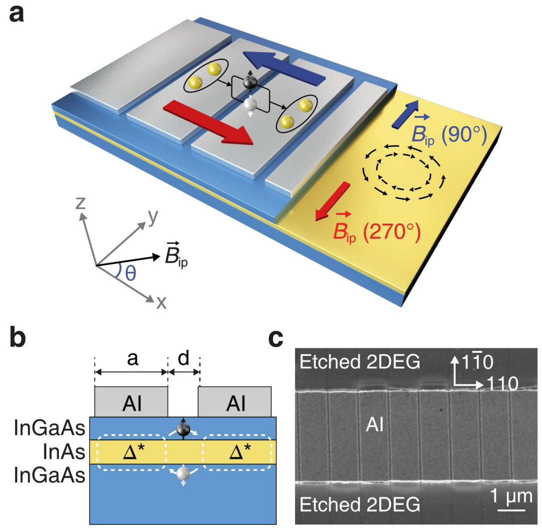
- broken inversion symmetry (P-symmetry)
- broken time-reversal symmetry (T-symmetry) – magnetic field

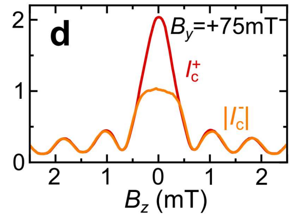
Josephson diode effect (experiment)

Supercurrent rectification and magnetochiral effects in symmetric Josephson junctions

Christian Baumgartner[1,8], Lorenz Fuchs[1,8], Andreas Costa[2], Simon Reinhardt[1], Sergei Gronin[3,4], Geoffrey C. Gardner[3,4], Tyler Lindemann[4,5], Michael J. Manfra[3,4,5,6,7], Paulo E. Faria Junior[2], Denis Kochan[2], Jaroslav Fabian[2], Nicola Paradiso[1]✉ and Christoph Strunk[1]

Nature Nanotechnology (2022)

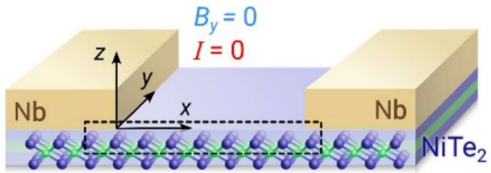


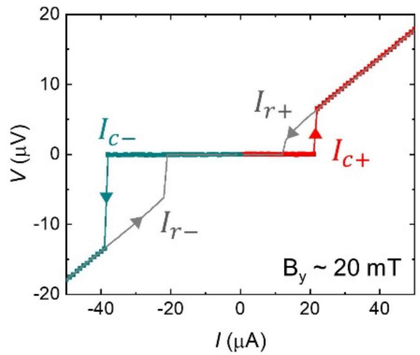


Josephson diode effect from Cooper pair momentum in a topological semimetal

Banabir Pal[1,4], Anirban Chakraborty[1,4], Pranava K. Sivakumar[1,4], Margarita Davydova[2,4], Ajesh K. Gopi[1], Avanindra K. Pandeya[1], Jonas A. Krieger[1], Yang Zhang[2], Mihir Date[1], Sailong Ju[3], Noah Yuan[2], Niels B. M. Schröter[1], Liang Fu[2]✉ and Stuart S. P. Parkin[1]✉

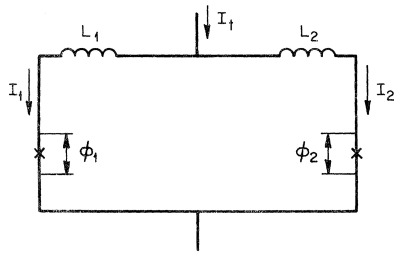
Nature Physics (2022)





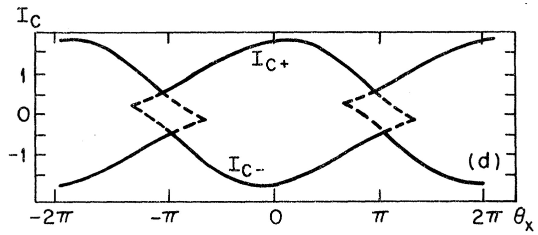
JDE due to inductive (self-field) effects

PHYSICAL REVIEW B VOLUME 6, NUMBER 3 1 AUGUST 1972



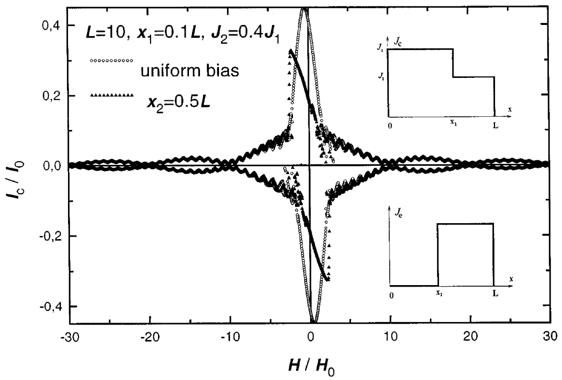
Quantum Interference Properties of Double Josephson Junctions

T. A. Fulton, L. N. Dunkleberger, and R. C. Dynes



PHYSICAL REVIEW B VOLUME 55, NUMBER 21 1 JUNE 1997-I

Fluxon dynamics in long Josephson junctions in the presence of a temperature gradient or spatial nonuniformity



V. M. Krasnov* V. A. Oboznov N. F. Pedersen

Department of Physics, Technical University of Denmark, DK-2800 Lyngby, Denmark
and Institute of Solid State Physics, Russian Academy of Sciences, 142432 Chernogolovka, Russia

Comment nature physics October 2023

Evolution of superconducting diodes

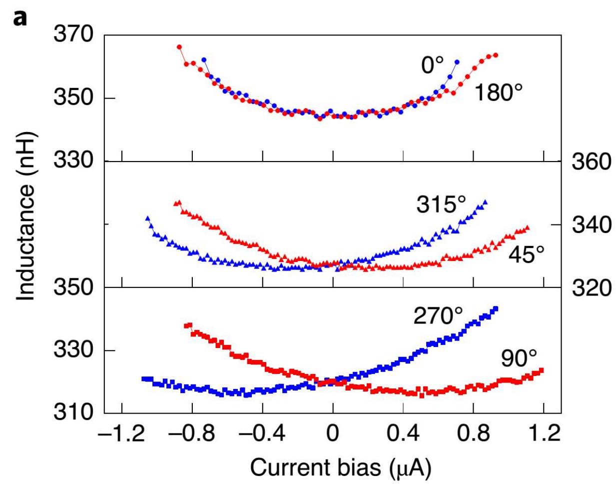
P. J. W. Moll & V. B. Geshkenbein

“Non-invasive” probes of the superconducting diode effect

Baumgartner *et al.*, Nat. Nanotech. (2022)

Josephson inductance in junctions:

$$V \propto \frac{d\varphi}{dt} \propto \frac{d\Phi}{dt} = L\frac{dI}{dt} \quad \longrightarrow \quad L \propto \frac{V}{\frac{dI}{d\varphi}\frac{d\varphi}{dt}} \quad \longrightarrow \quad L = \frac{\Phi_0}{2\pi}\left(\frac{dI}{d\varphi}\right)^{-1}$$



Diode effect: $L(-I) \neq L(I)$, and the minimum is shifted from $I = 0$

Kinetic inductance in bulk:

$$L_k \propto \left(\frac{dI}{dq}\right)^{-1}$$

Similarly to magnetoelectric anisotropy of resistance:

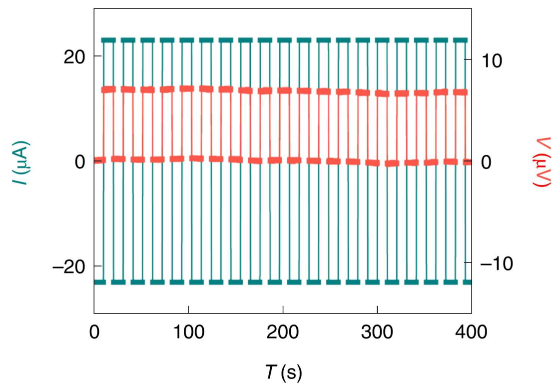
$$L(I) = L_0\big(1 + \gamma_L \hat{\mathbf{z}}(\mathbf{B} \times \mathbf{I})\big)$$

Superconducting diode effect: Why so popular?

- Interesting fundamental effect depending on symmetries and, hence, probing them

- Possible applications:
  - voltage rectifiers
  - filters
  - current converters

- Diverse physical platforms and physical mechanisms:
  - ❑ bulk systems and Josephson junctions
  - vortices
  - stray fields of magnets
  - exchange field and spin-orbit coupling
  - topological materials
  - spatial symmetry breaking either on atomic level (NCS crystals) or macroscopically by heterostructure design
  - self-field effects in long Josephson junctions and in SQUIDs
  - SQUIDs with higher Josephson harmonics

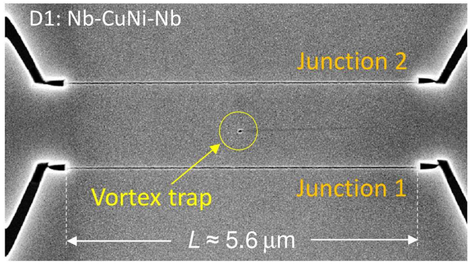


Pal *et al.*, Nature Physics (2022)

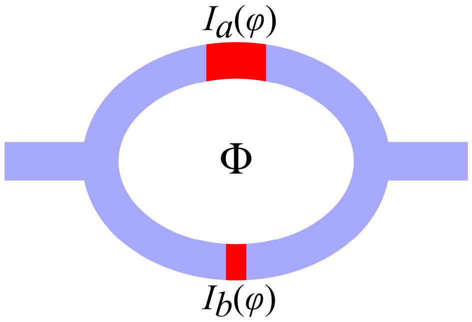
Demonstration of a superconducting diode-with-memory, operational at zero magnetic field with switchable nonreciprocity

Taras Golod[1] & Vladimir M. Krasnov[1]

NATURE COMMUNICATIONS | (2022)13:3658



1a. Josephson diode effect in asymmetric SQUIDs with higher harmonics



$$\varphi_a - \varphi_b = \phi, \qquad \phi = 2\pi \frac{\Phi}{\Phi_0}$$

CRP of the SQUID: $I_s(\varphi, \phi) = I_a(\varphi_a) + I_b(\varphi_b) = I_a(\varphi + \phi/2) + I_b(\varphi - \phi/2)$

Sinusoidal case: $I_s(\varphi, \phi) = I_{a1} \sin(\varphi + \phi/2) + I_{b1} \sin(\varphi - \phi/2) = \boxed{I_1(\phi) \sin(\varphi + \gamma)}$

where $I_1(\phi) = \sqrt{I_{a1}^2 + I_{b1}^2 + 2 I_{a1} I_{b1} \cos\phi}$

$$\tan\gamma = \frac{I_{a1} - I_{b1}}{I_{a1} + I_{b1}} \tan\frac{\phi}{2}$$

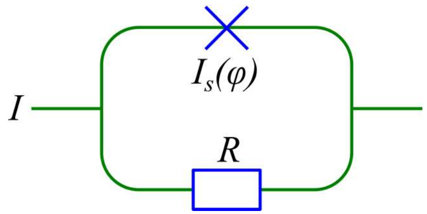
Critical currents in two directions: $I_c^+ = I_c^-$

More generally, this is so

(a) in symmetric SQUIDs with arbitrary $I_a(\varphi) = I_b(\varphi)$

(b) when $I_a(\varphi)$ and $I_b(\varphi)$ contain the same single harmonic (with arbitrary amplitudes)

Resistively-shunted junction (RSJ) model



$$\frac{V}{R} + I_s(\varphi) = I, \qquad V = \frac{\hbar}{2e}\frac{\partial\varphi}{\partial t}$$

$I_s(\varphi) = I_1 \sin\varphi$ — sinusoidal current-phase relation (CPR)

Units: $\omega_0 = 2eI_1R/\hbar, \quad \tau = \omega_0 t, \quad j = I/I_1, \quad v = V/I_1R$

Resistively-shunted junction (RSJ) model:

$$v = \dot{\varphi}$$

$$\dot{\varphi} + \sin\varphi = j$$

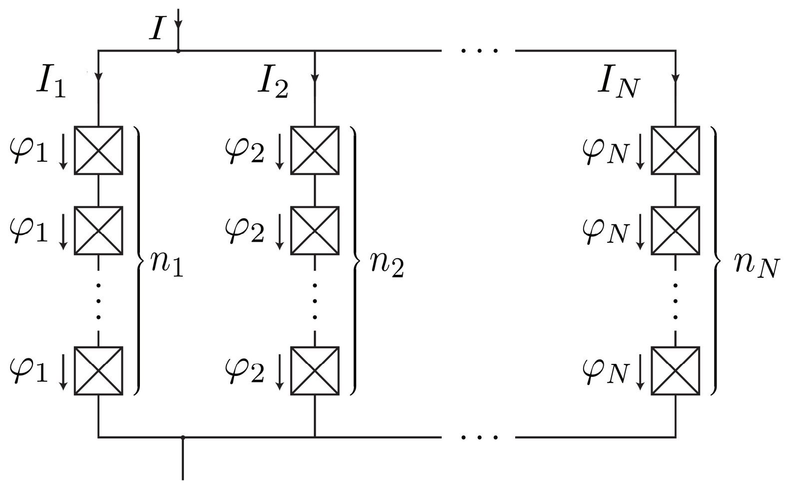
[Aslamazov, Larkin, Ovchinnikov (1968),
Stewart (1968),
McCumber (1968)]

Higher Josephson harmonics

$$I_s(\varphi) = \sum_{n=1}^{\infty} I_n \sin n\varphi$$ — nonsinusoidal CPR with higher harmonics

- higher orders wrt barrier transparency
- point contacts
- SNS junctions
- *SFS junctions*
- CPR engineering
- etc.

[Haenel, Can, arXiv(2022)]

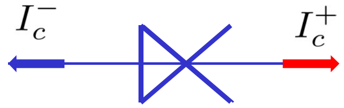


RSJ equation: $\dot{\varphi} + J(\varphi) = j, \qquad J(\varphi) = I_s(\varphi)/I_1$

General relation: $\overline{v} = \nu$

Asymmetric higher-harmonic SQUID as a Josephson diode

General case: $I_c^+ \neq I_c^-$



"Minimal model":

$$I_a(\varphi) = I_{a1} \sin\varphi$$

$$I_b(\varphi) = I_{b1} \sin\varphi + I_{b2} \sin 2\varphi$$

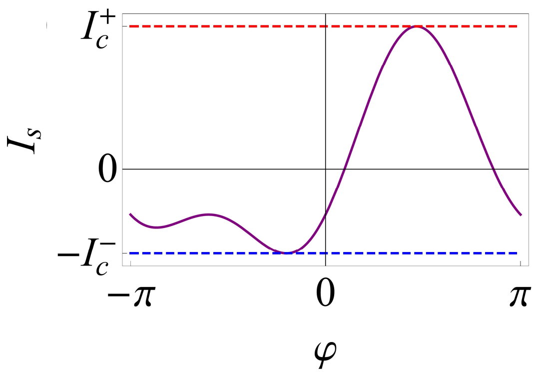
Effective Josephson junction with

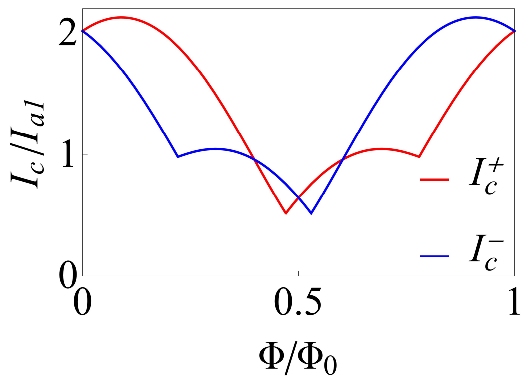
$$J(\varphi) = \sin\varphi + A \sin(2\varphi - \tilde{\phi})$$

where

$$A(\phi) = I_{b2}/I_1(\phi), \quad \tilde{\phi} = \phi + 2\gamma(\phi)$$

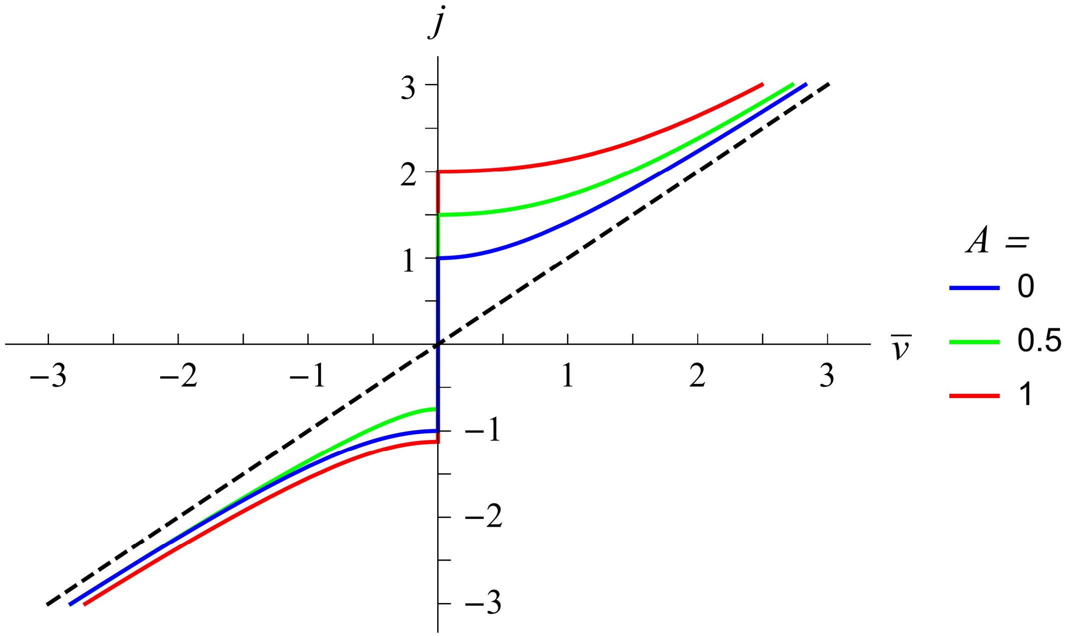
Analytics in limiting cases: diode effect at $A \sin\tilde{\phi} \neq 0$





— asymmetry of critical currents

Asymmetry of the CVC

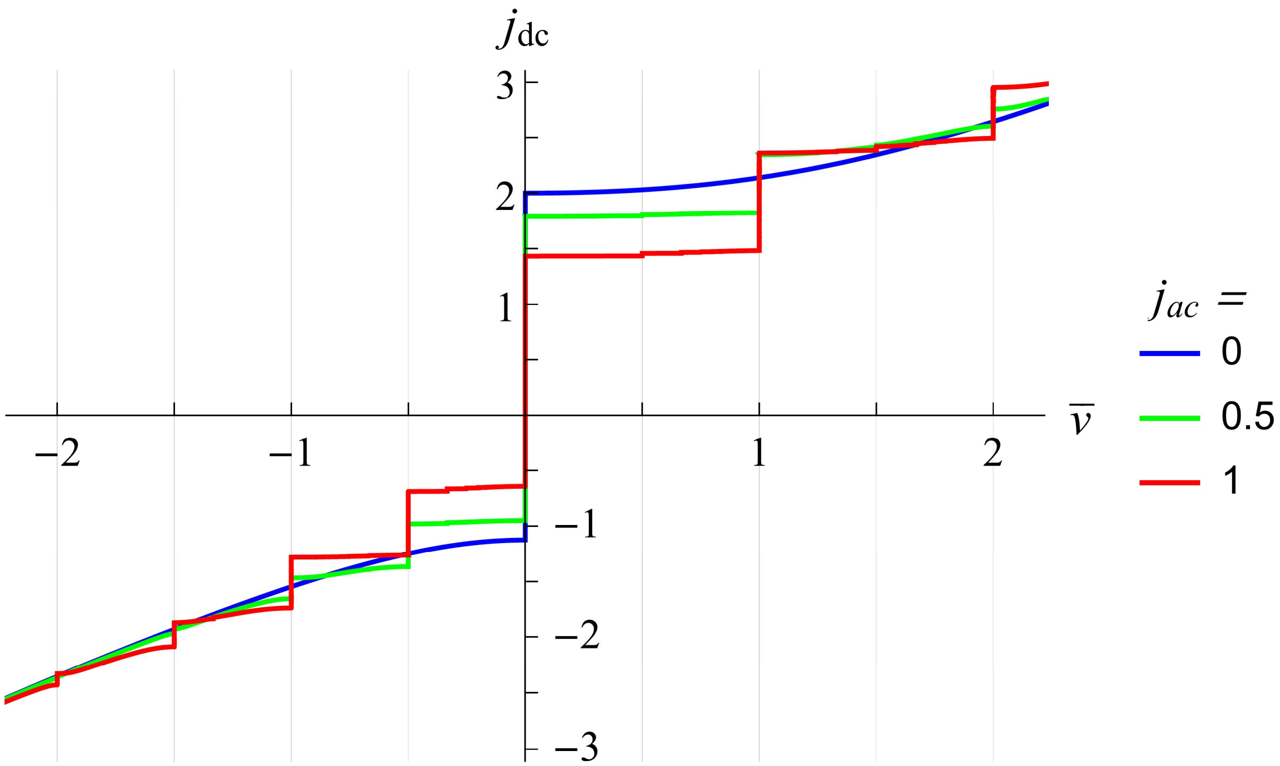


Diode effect: $j_c^+ \neq j_c^-$

$$j(-\overline{v}) \neq -j(\overline{v})$$

Asymmetry of the Shapiro steps

$$j = j_{\mathrm{dc}} + j_{\mathrm{ac}} \cos \omega t$$



Diode effect: $j_c^+ \neq j_c^-$

$$j_{\mathrm{dc}}(-\overline{v}) \neq -j_{\mathrm{dc}}(\overline{v})$$

Conclusions-1a

Josephson diode effect in asymmetric SQUIDs with higher Josephson harmonics.

Minimal model: $J(\varphi) = \sin\varphi + A\sin(2\varphi - \tilde{\phi})$, at $A\sin\tilde{\phi} \neq 0$

- Asymmetric critical current: $I_c^+ \neq I_c^-$
- Asymmetric current-voltage characteristic: $I(-V) \neq -I(V)$
- Asymmetric integer and fractional Shapiro steps (current-driven regime),

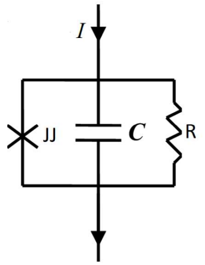
  $$\overline{V} = \left(\frac{n}{k}\right)\frac{\hbar\omega}{2e}$$

- Efficiency and polarity of the diode effect depend on the external magnetic flux

[1a] Ya.V. Fominov, D.S. Mikhailov, *Asymmetric higher-harmonic SQUID as a Josephson diode*, Phys. Rev. B **106**, 134514 (2022).

1b. Effects of capacitance and temperature

RSJ model $\Longmapsto$ RCSJ model (added capacitance and thermal fluctuations):

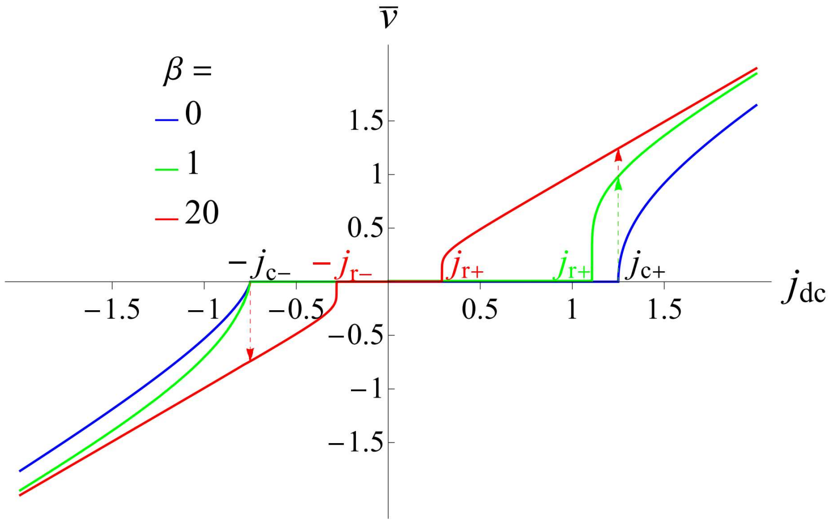


$$\frac{d^2\varphi}{d\tilde{\tau}^2} + \varepsilon\frac{d\varphi}{d\tilde{\tau}} + J(\varphi) = j_{\mathrm{dc}} + j_{\mathrm{ac}}\cos(\tilde{\omega}\tilde{\tau} + \delta) + \xi(\tilde{\tau})$$

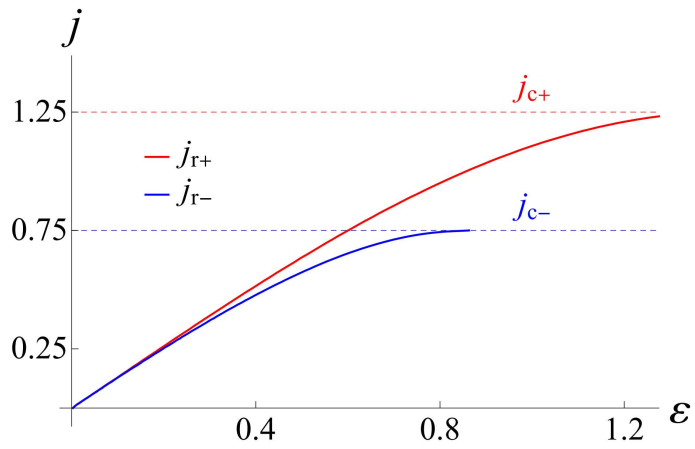
$$\varepsilon = 1/\sqrt{\beta}, \quad \beta = (2e/\hbar)I_1R^2C, \quad \tilde{\tau} = \omega_p t, \quad \omega_p = \sqrt{2eI_1/\hbar C}$$

McCumber parameter

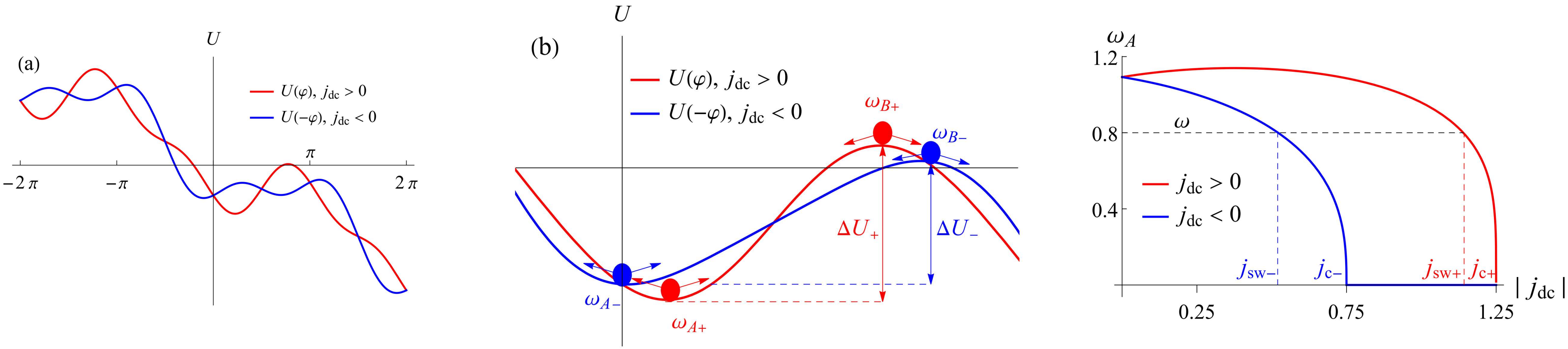
Effects of capacitance



- Asymmetry in the R state is generally suppressed by capacitance (both without and with external ac irradiation)
- New effect: asymmetry of the retrapping currents $j_{r\pm}$
- Single-sides hysteresis



Asymmetry of the switching currents (S to R state)



- Effect of external ac irradiation on the S state:
  Josephson plasma resonances at $\omega = \omega_{A\pm}(j_{sw\pm})$

$$j_{\mathrm{sw}\pm} = \sqrt{1-\tilde{\omega}^4} + \frac{2A\tilde{\omega}^6 \cos\tilde{\phi}}{\sqrt{1-\tilde{\omega}^4}} \pm A(1+2\tilde{\omega}^4)\sin\tilde{\phi}$$

Effects of thermal fluctuations

$$\beta\ddot{\varphi} + \dot{\varphi} + \sin\varphi + A\sin(2\varphi - \tilde{\phi}) = j_{\rm dc} + \xi(\tau)$$

$$\langle \xi(\tau)\xi(\tau')\rangle = 2\theta\delta(\tau - \tau'), \ \text{where } \theta = 2eT/\hbar I_1 = T/E_J$$

- Generalization of the Ambegaokar-Halperin method:
  Fokker-Planck equation for the distribution function
  in the low-temperature limit $\theta \ll \Delta U_{\pm}$ (i.e., escape time $\gg$ sliding time)

Zero capacitance, asymmetric CVC:

$$\langle \overline{v}_{\pm}(j_{\rm dc})\rangle = \pm 2\omega_{A\pm}\omega_{B\pm}\sinh\left(\frac{j_{\rm dc}\pi}{\theta}\right)\exp\left(\frac{-\Delta U_{\pm} - j_{\rm dc}\pi}{\theta}\right)$$

Nonzero capacitance, asymmetric escape rates:

$$\tilde{\tau}_{l\pm}^{-1} = \frac{\tilde{\omega}_{\rm att\pm}}{2\pi}\exp\left(-\frac{\Delta U_{\pm}}{\theta}\right)$$

Effects of thermal fluctuations: switching currents

Slowly increasing current: $j_{\rm dc}(\tilde{\tau}) = a\tilde{\tau}, \quad a\tilde{\tau}_{l\pm} \ll 1$

Switching: probability to remain in the potential well = 1/2

$$j_{\rm sw\pm} = j_{c\pm} - \begin{cases} \left(\dfrac{\theta}{u_{c\pm}} \ln \dfrac{2\theta\omega_{c\pm}^2}{(6\pi \ln 2)\varepsilon a u_{c\pm}}\right)^{2/3}, & \varepsilon \gg 1 \\ \left(\dfrac{\theta}{u_{c\pm}} \ln \dfrac{2\theta\omega_{c\pm}}{(6\pi \ln 2) a u_{c\pm}}\right)^{2/3}, & \varepsilon \ll 1 \end{cases}$$

Conclusions-1b

Capacitance:

- Generally, capacitance suppresses the diode effect in the R state

At the same time, qualitatively new effects:

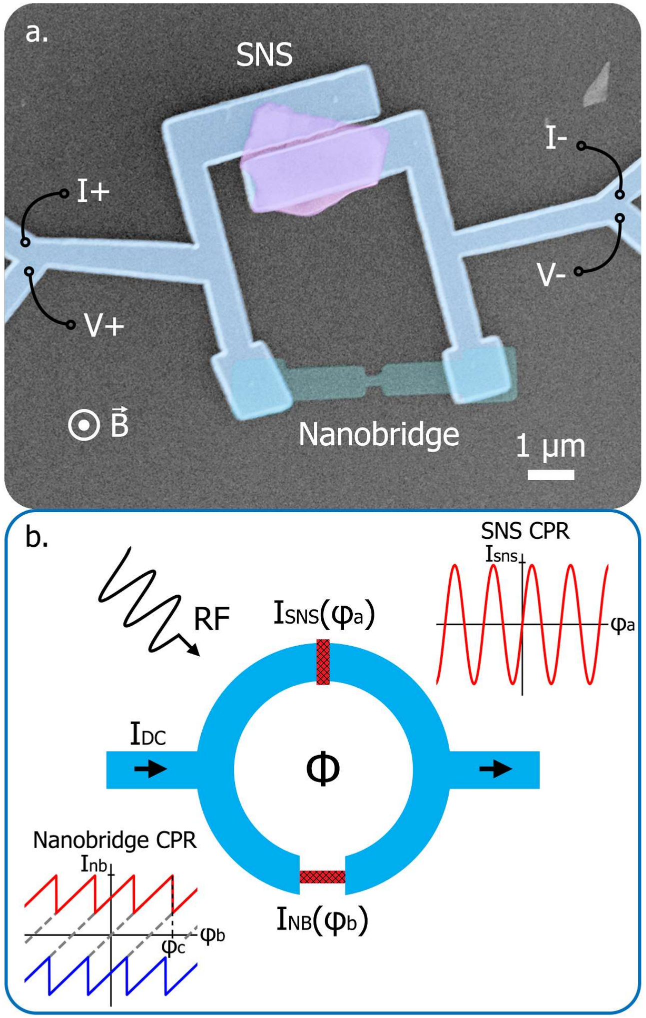
- Asymmetry of the retrapping currents
- Single-sided hysteresis
- Asymmetric switching currents due to the Josephson plasma resonances

Thermal fluctuations:

- Exponentially asymmetric CVC below the critical currents
- Asymmetry of the thermal switching currents

[1b] **G.S. Seleznev**, Ya.V. Fominov, *Influence of capacitance and thermal fluctuations on the Josephson diode effect in asymmetric higher-harmonic SQUIDs*, Phys. Rev. B **110**, 104508 (2024).

2. Diode effect in Shapiro steps in an asymmetric SQUID with a superconducting nanobridge



Center for Advanced Mesoscience & Nanotechnology

CPR of the SQUID with nanobrigde:

$$I_s(\varphi) = I_{\rm sns}(\phi)\sin(\varphi+\phi) + I_{\rm NB}(\varphi)$$

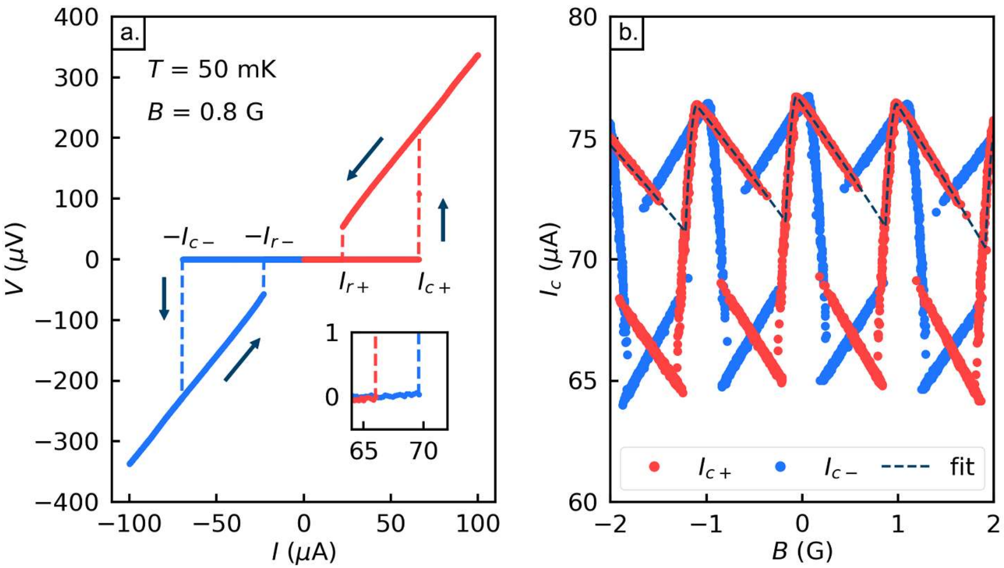
$$\frac{I_s(\varphi)}{I_{\rm nb}} = A_{0\pm} + \sum_{k=1}^{\infty} A_{k\pm}\sin\big(k(\varphi+\delta_{k\pm})\big)$$

Asymmetry of the first Josephson harmonic at $\sin\varphi_c \sin\phi \neq 0$:

$$A_{1\pm} = \sqrt{\left(\frac{I_{\rm sns}(\phi)}{I_{\rm nb}}\right)^2 + \frac{4}{\varphi_c^2} - \frac{4I_{\rm sns}(\phi)}{I_{\rm nb}\varphi_c}\cos\left(\phi \pm \varphi_c\right)}$$

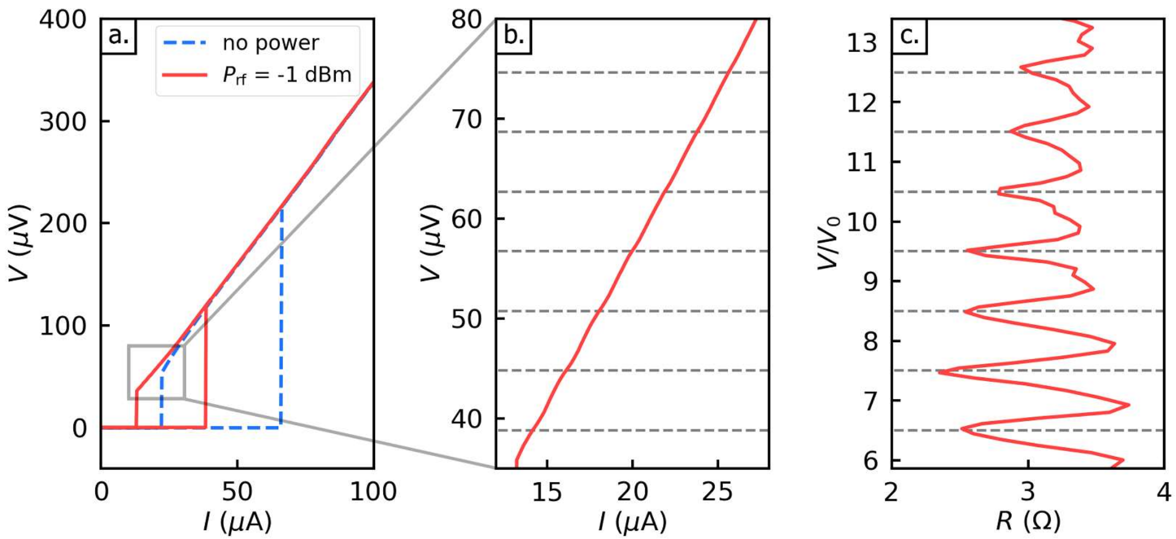
— leads to JDE!

dc and ac measurements

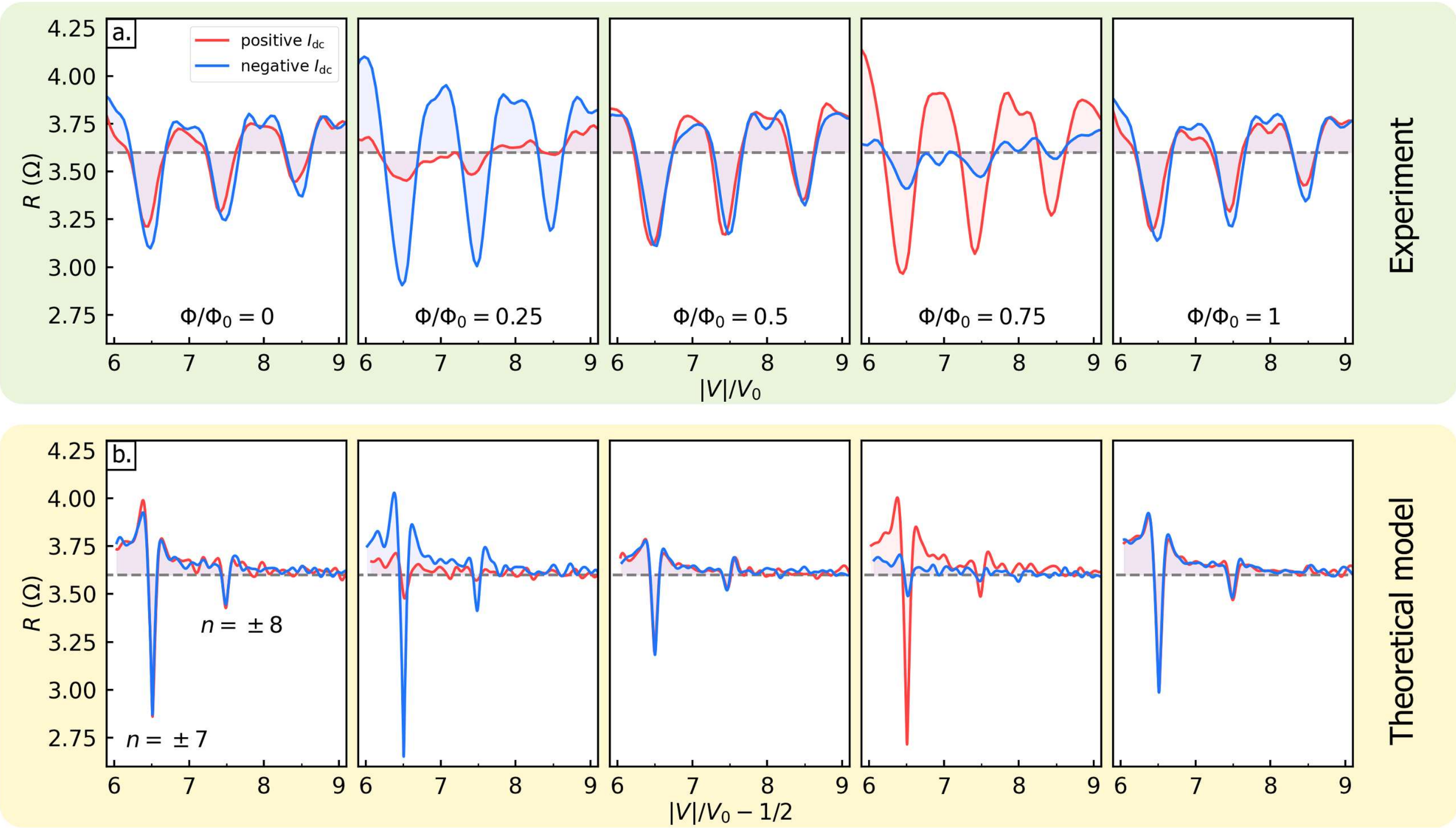


— dc measurements: asymmetry of the critical currents

— ac measurements: Shapiro steps



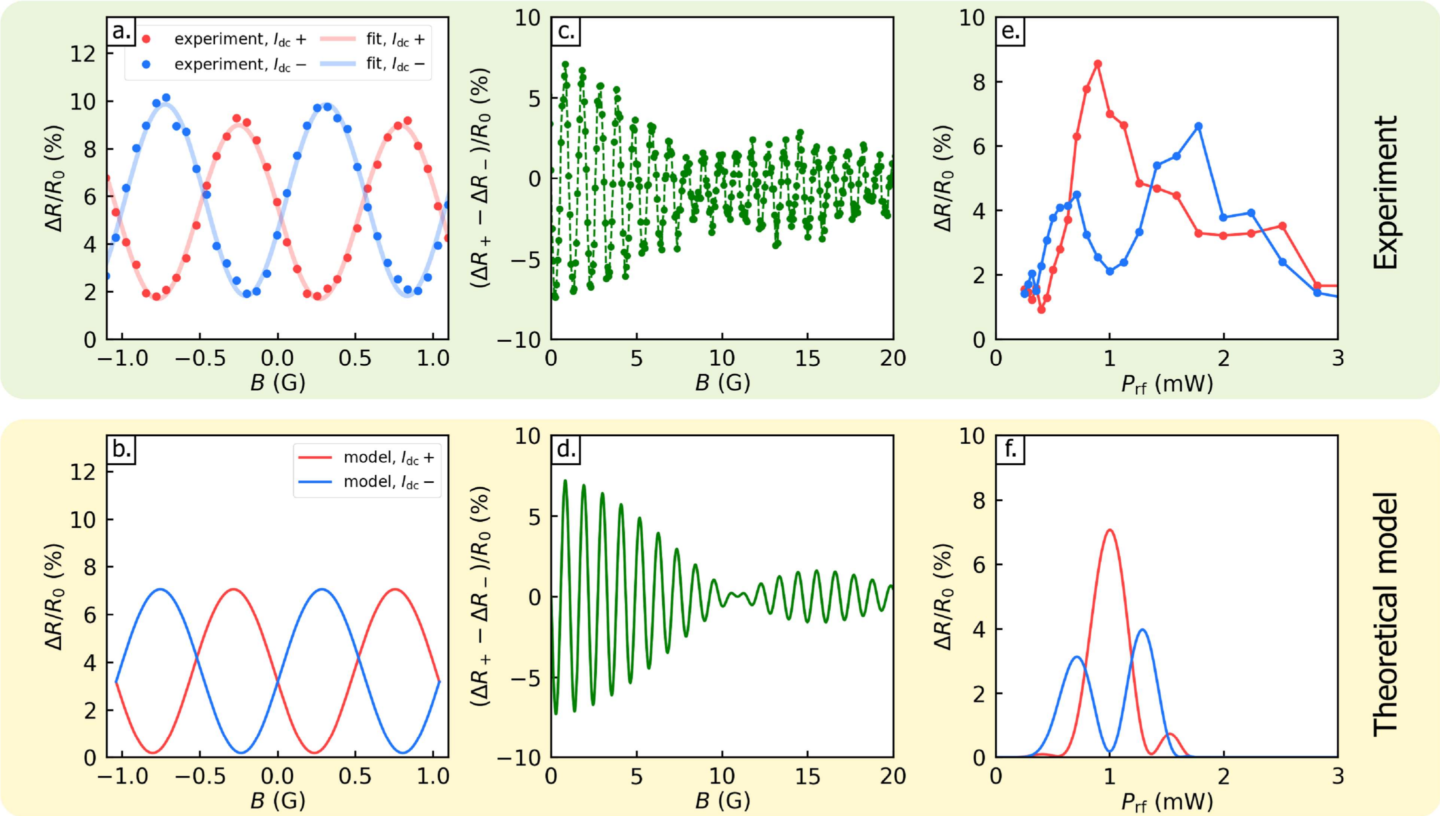
Asymmetry of Shapiro steps



Theory: RSJ model with thermal noise: $\langle \xi(\tau)\xi(\tau') \rangle = 2\mathcal{T}\delta(\tau - \tau')$

$$\mathcal{T} = 2eT/\hbar I_{\mathrm{nb}}$$

Analysis of the Shapiro steps



$$\mathcal{T}_\pm = \mathcal{T}/A_{1\pm}J_n(j_{\rm ac}/\omega)$$

Theory: single-harmonic CPR.

Analytics at the centers of the Shapiro steps:

$$\frac{R_\pm}{R_0} = I_0^{-2}\left(1/\mathcal{T}_\pm\right) = \begin{cases} (2\pi/\mathcal{T}_\pm)\exp(-2/\mathcal{T}_\pm), & \mathcal{T}_\pm \ll 1 \\ (1-1/2\mathcal{T}_\pm^2), & \mathcal{T}_\pm \gg 1 \end{cases}$$

Dependence on power of microwave irradiation: account of energy balance.

Conclusions-2

Asymmetric SQUID with nanobridge:

- Josephson diode effect manifestations in the critical currents and Shapiro steps

- New mechanism: asymmetry of the first Josephson harmonic in the effective SQUID's CPR due to (linear) multivalued CPR of the nanobridge:

  $A_{1+} \neq A_{1-}$ at $\sin \varphi_c \neq 0$

[2] D.S. Kalashnikov, **G.S. Seleznev**, A. Kudriashov, Y. Babich, D.Yu. Vodolazov, Ya.V. Fominov, V.S. Stolyarov, *Diode effect in Shapiro steps in an asymmetric SQUID with a superconducting nanobridge*, in preparation.

3. Superconducting orbital diode effect in SN bilayers

PHYSICAL REVIEW B **108**, 094517 (2023)

Finite momentum superconductivity in superconducting hybrids: Orbital mechanism

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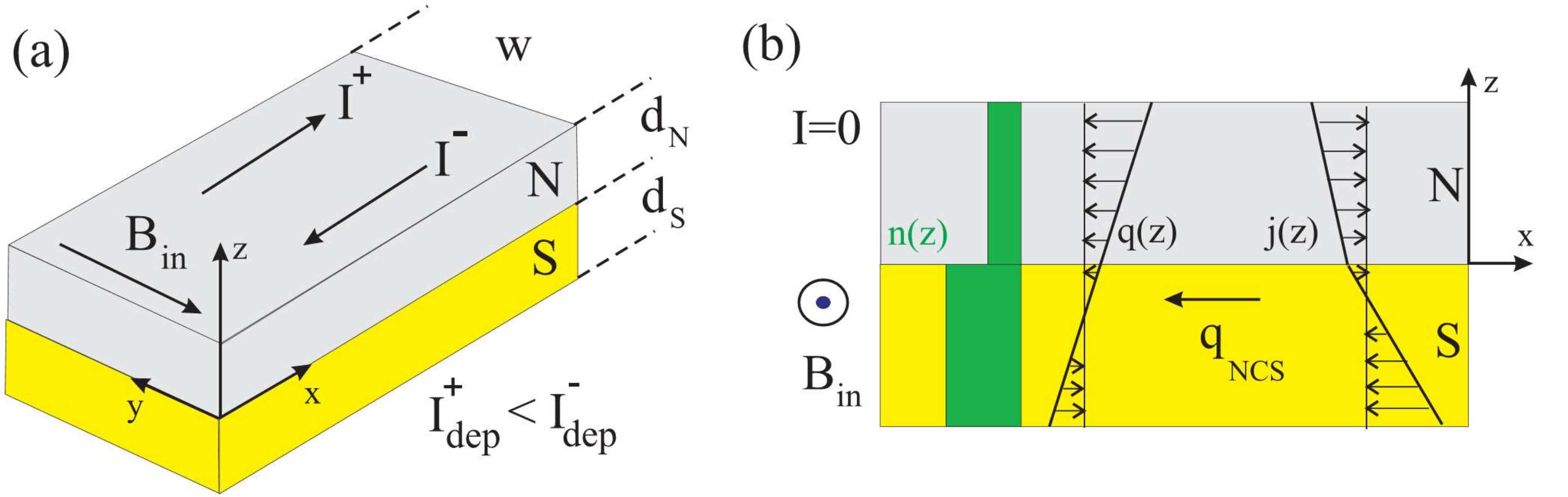


Symmetry-breaking vector: $[\nabla n \times \mathbf{B}]$

Numerics in the case of transparent interface
+ experiment

Analytics + interface effect

Analytics: dirty-limit theory (Usadel equations) in the case of weak inhomogeneity, development of

PHYSICAL REVIEW B, VOLUME 63, 094518 (2001)

Superconductive properties of thin dirty superconductor-normal-metal bilayers

Ya. V. Fominov and M. V. Feigel'man

Weakly inhomogeneous case:

1. Thin bilayer $d \ll \xi$ + arbitrary interface transparency

2. Thick bilayer $d \geq \xi$ + opaque interface

Condition: $$\frac{d^2}{\xi^2} \ll \max\left(1, \frac{R}{R_0}\right)$$

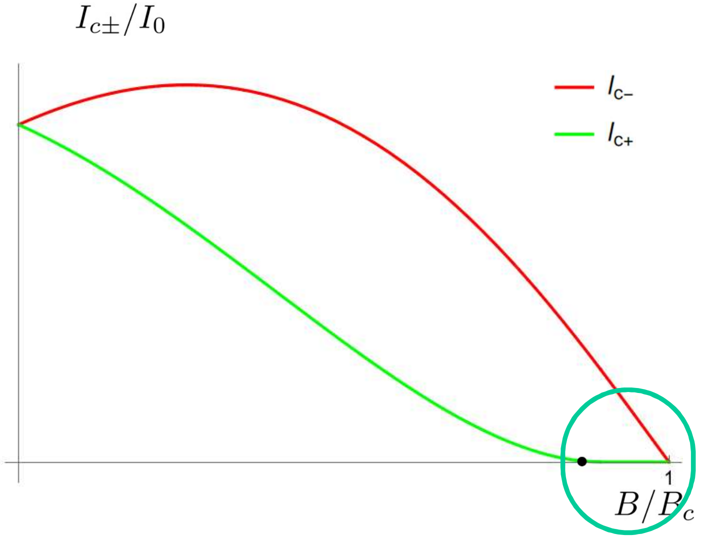
$$R_0 = R_q \frac{\delta_S + \delta_N}{|\Delta|}$$

R — interface resistance

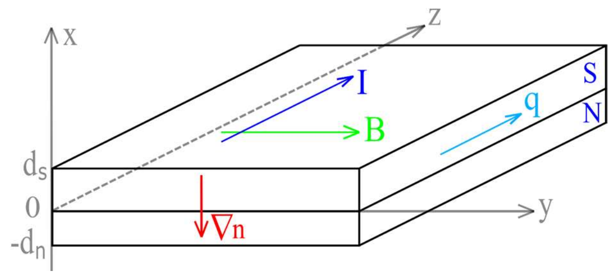
R_q — quantum resistance

$\delta_{S,N} = (\nu_{S,N} V_{S,N})^{-1}$ — level spacing

Supercurrent



$$\frac{I(q,B)}{I_0} = \frac{n_{\text{eff}}(q,B)}{n_0}\,\frac{q}{q_c} + k\left|\frac{\Delta(q,B)}{\Delta_0}\right|^2 \frac{B}{B_c}$$



— full diode effect is achievable

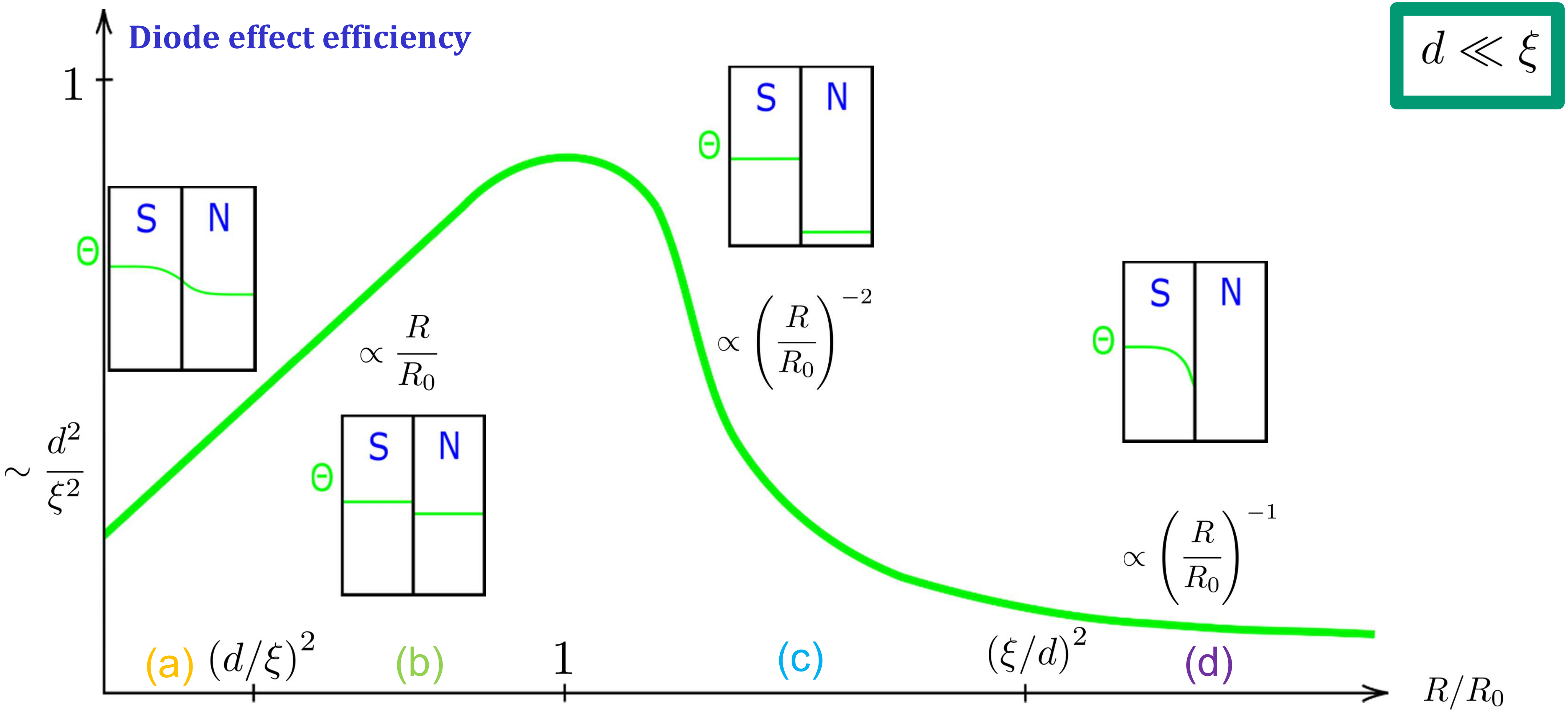
n_0, Δ_0 — superfluid density and order parameter at $T = 0$ and $B = 0$

q_c, B_c — critical momentum and magnetic field

I_0 — characteristic current value (at $n_{\text{eff}} = n_0$, $q = q_c$, $B = 0$)

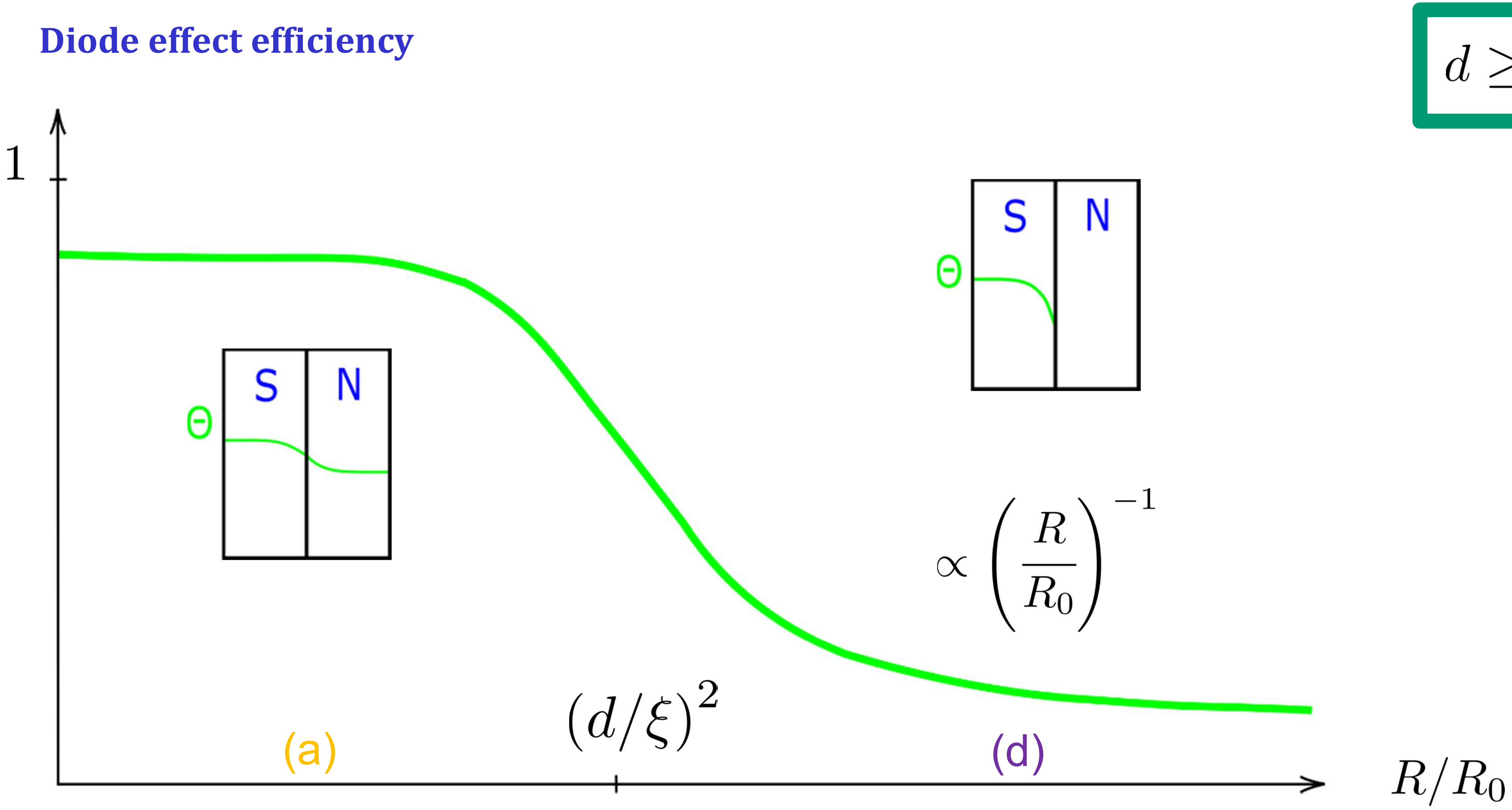
$k \ll 1$ — parameter of weak inhomogeneity

Interface effect: thin bilayer



Interface effect: thick bilayer

$d \geq \xi$



Conclusions-3

Orbital diode effect in SN bilayers:

- Analytics in weakly inhomogeneous cases $d^2/\xi^2 \ll \max(1, R/R_0)$
- Dependence of critical currents and diode efficiency on magnetic field: absolute visibility at $B \to B_c$
- Effect of transparency: nonmonotonic diode efficiency in the limit of $d \ll \xi$ with maximum at $R \sim R_0$

[3] **Yu.A. Dmitrievtsev**, Ya.V. Fominov, *Superconducting orbital diode effect in SN bilayers*, in preparation.

Conclusions

Superconducting diode effect:

- Fundamental effect revealing symmetries of the system

- Diverse physical platforms and physical mechanisms

❑ Work supported by the Russian Science Foundation (Grant No. 24-12-00357)